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PROBABILITY, RANDOM VARIABLE AND RANDOM PROCESS

THEORY

1. INTRODUCTION

In the analysis of communcation system, we often encounter, random signals, whose behaviour connot be predicted exactly.

Th output of information sources and noise are random in nature. Hence random signal cannot be expressed as explicit function of time.

However when examine over a long period, randon signal may exhibit certain properties which is regular and can be described in terms of probabilities and statistical averages. Hence our objective is to develope probabilistic models of random signals.

2. RANDOM EXPERIMENT

All the possible outcome are known in advance but the exact outcome is unknown.

3. SAMPLE SPACE (Ω) OR (S)

Set of all possible outcomes.

4. EVENT (E)

It is set of desired outcome.

Example: $\Omega = \{1, 2, 3, 4, 5, 6\}$

E = Number of odd terms

$E = \{1, 3, 5\}$

$E = \{1, 3, 5\}$

Example : Tossing a fair coin

$\rightarrow$ Random experiment

$\rightarrow$ Tossing a fair coin

sample space ‘Ω’ = {H, T}

➢ **Events :**

$E_1 = \{H\}$

$E_2 = \{T\}$

$E_3 = \{H, T\}$

$E_4 = \{\text{gold}\}$

Probability of occurrence of E_1

$$P\{E_1\} = \frac{1}{2}$$

Probability of occurrence of E_2

$$P\{E_2\} = \frac{1}{2}$$

Probability of occurrence of E_3

$$P\{E_3\} = 1 = \text{sure event}$$

Probability of occurrence of E_4

$$P\{E_4\} = 0 = \text{Null event}$$

➢ **Null Event :**

If the probability of occurrence of any event is 0, then it is called as null event.

➢ **Sure Event:**

If the he probability of occurrence of certain event is 1, then it is called as sure event.

$$P\{E_1 \cup E_2\} = 1 \rightarrow \text{exhaustive event}$$

$$P\{E_1 \cap E_2\} = \phi \rightarrow \begin{cases} \rightarrow \text{Dis-joint event} \\ \rightarrow \text{Mutually-exclusive event} \end{cases}$$

Example: Random experiment : Throwing a fair dice.

Sample space $\Omega = \{1, 2, 3, 4, 5, 6\}$

Events:

E_1 = occurrence of '1' = $\{1\}$

E_2 = occurrence of odd numbers = $\{1, 3, 5\}$

E_3 = occurrence of even numbers = $\{2, 4, 6\}$

E_4 = occurrence of square numbers = $\{1, 4\}$

E_5 = occurrence of negative numbers = $\{\phi\}$

E_6 = occurrence of natural numbers = $\{1, 2, 3, 4, 5, 6\}$

$$P\{E_1\} = \frac{1}{6}$$

$$P\{E_2\} = \frac{3}{6}$$

$$P\{E_3\} = \frac{3}{6}$$

$$P\{E_4\} = \frac{2}{6}$$

$$P\{E_5\} = 0$$

$$P\{E_6\} = 1$$

Probability of finding any event can be mathamatically realised by

$$P(E) = \lim_{x\to\infty}\left\{\frac{N(E)}{N}\right\}$$

Where, N denotes the number of times a ramdom experiment is performed and

$$N(E) = \text{number of times event occurs}$$

$$P\{E_2 \cap E_4\} = \{1\} = \frac{1}{6}$$

$$P\{E_2 \cup E_4\} = \{1, 3, 4, 5\} = \frac{4}{6}$$

$$\left.\begin{array}{l} P(E_2 \cap E_3) = 0 \\ P(E_2 \cup E_3) = 1 \end{array}\right\} \begin{array}{l} \rightarrow \text{mutually exclusive} \\ \quad \& \\ \rightarrow \text{exhaustive in nature} \end{array}$$

From set theory,

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

If event is mutually exclusive

$$P\{A \cap B\} = 0$$

$$\boxed{P\{A \cup B\} = P(A) + P(B)}$$

If event is exhaustive

$$\boxed{P\{A \cup B\} = 1}$$

$\therefore$ If event is mutually exclusive as well as exhaustive then

$$P(A \cup B) = P(A) + P(B) = 1$$

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$$

Example: A box contains 5 black and 5 red balls, two balls are randomly picked one after another from box without replacement, the probability for both balls being red is?

Solution:

$$E_1 = \{5 - R, 5 - B\} \rightarrow \text{without replacement}$$

$$E_2 = \{4 - R, 5 - B\}$$

$$P = \frac{5}{10} \times \frac{4}{9} = \frac{2}{9}$$

Note: (a) Permutation $\quad P = {}^nP_r = \dfrac{n!}{(n-r)!}$

It is used where sequence is important.

(b) Combination: $C = n_{c_r} = \dfrac{n!}{r!(n-r)!}$

It is used when order is not important.

5. BINOMIAL THEOREM:

Success and failure are mutually exclusive and exhaustive events

Let us assume

Probability of success = p

Probability of failure = q

$$p + q = 1$$

Example: Let a fair coin is tossed n times and head (success) occurs r times so tail (failure) occurs (n–r) times.

Solution: Probability of 'r' success in 'n' trials

$$P(r) = {}^nC_r\, p^r q^{n-r} = \text{Binomial theorem}$$

6. JOINT PROBABILITY:

$$P(A,B) = P\left(\frac{A}{B}\right).\ P(B)$$

$$= P\left(\frac{B}{A}\right).\ P(A)$$

P(A,B) = Joint probability

i.e probability of occurence of A and B.

P(A/B) = Probability of occurence of A provided B is already occurred.

P(B/A) = Probability of occurence of B provided A is already occurred.

7. CONDITIONAL PROBABILITY.

We define a turn called as & satistically independent event = occurence of event 'A' does not depend on occurence of event 'B' and vice-versa.

Mathematically,

$$P\left(\frac{A}{B}\right) = P(A)$$

$$\therefore \quad P(A,B) = P(A).P(B)$$

Example: Three boxes B_1,B_2, B_3. B_1 contains 7 red and 3 white balls. B_2 contains 4 red and 6 white balls. B_3 contains 10 red balls. A ball is drawn from box B_2. What is the probability that it is red. All the three boxes can be seleced with equal probability.

Solution:

B_1 = 7R, 3W

B_2 = 4R, 6W

B_3 = 10 R

$$P(B_1) = P(B_2) = P(B_3) = \frac{1}{3}$$

$$P(R, B_2) = ?$$

$$P(R,B_2) = P\left(\frac{R}{B_2}\right)\times P(B_2)$$

$$P\left(\frac{R}{B_1}\right) = \frac{7}{10}$$

$$P\left(\frac{R}{B_2}\right) = \frac{4}{10}$$

$$P\left(\frac{R}{B_3}\right) = 1$$

$$P\left(\frac{W}{B_1}\right) = \frac{3}{10}$$

$$P\left(\frac{W}{B_2}\right) = \frac{6}{10}$$

$$P\left(\frac{W}{B_3}\right) = 0$$

$$P(R,B_2) = P\left(\frac{R}{B_2}\right)\times P(B_2)$$

$$= \frac{4}{10}\times\frac{1}{3}$$

$$= \frac{4}{30} = \frac{2}{15}$$

Example: As per the data given in the above question. A ball is selected randomly and its comes out to be red. What is the probability that is belong to box B_2?

$$P(B_2/R) = ?$$

Solution:

$$P(R,B_2) = P\left(\frac{B_2}{R}\right).P(R)$$

$$P\left(\frac{B_2}{R}\right) = \frac{P(R,B_2)}{P(R)}$$

$$P\left(\frac{B_2}{R}\right) = \frac{P(R, B_2)}{P(R)}$$

$$= \frac{\frac{4}{10}\times\frac{1}{3}}{\frac{21}{30}} = \frac{4}{21} = \frac{P\left(\frac{B_2}{R}\right).P(B_2)}{P(R)}$$

8. RANDOM VARIABLE

Random experiment: Tossing a coin three times.

Event : Getting heads

S = {HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}= sample space

$X = \{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8\}$

X = Random variable:

x = Values that this Random variable is going to table

X = {3, 2, 2, 1, 2, 1, 1, 0}

A random variable signifies a rule by which a real number is assigned to each possible outcome of an experiment. Hence random variable is a mapping from sample space to real axis.

Note: *In sample space domain, we talk interms of probability of occurrence but when we come to real axis we define terms like cumulative density function C.D.F and P.D.F.*

(i) ***Probability Mass Function :*** $P_X(x)$

X = Random variable

x = Values taken by random variable

Mathematically,

$$P_X(x) = P(X = x_i)$$

(ii) ***Cumulative Distribution Function :*** $F_X(x)$

$$F_X(x) = P(X \le x)$$

Properties:

(1) $F_X(\infty) = 1$

(2) $F_X(-\infty) = 0$

(3) $$F_X(x) = \begin{cases} 0 & ; x < x_1 \\ \sum_{i=1}^{k} P(x = x_i); & x_1 \le x \le x_k \\ 1 & ; x > x_k \end{cases}$$

(4) $0 \le F_X(x) \le 1$

(5) If $x_2 > x_1$

then $F_X(x_2) > Fx(x_1)$

$F_X(x)$ is monotone and non-decreasing

(6) $P(x_1 < X \le x_2) = F_X(x_2) - F_X(x_1)$

(7) $P(X = x_0) = F_X\left(x_0^+\right) - F_X\left(x_0^-\right)$

(iii) Probability Density Function (PDF) $[f_X(x)]$

Mathematically,

$$f_X(x) = \frac{d}{dx} F_X(x)$$

$$F_X(x) = \int_{-\infty}^{x} f_X(x)dx$$

PDF is monotone & non decreasing

Area under PDF = 1

i.e. $$\int_{-\infty}^{x} f_X(x)dx = 1$$

$$P(x_1 < X \le x_2) = F_X(x_2) - F_X(x_1)$$

$$P(x_1 < X \le x_2) = \int_{x_1}^{x_2} f_X(x)dx$$

9. STATISTICAL AVERAGE:

(i) n^{th} moment of a random variable

$$E[X^n] = \int_{-\infty}^{\infty} x^n f_x(x)dx$$

If $n = 1$ $\quad E[x] = \int_{-\infty}^{\infty} x f_x(x)dx$ mean value or average value = M_1

If $n = 2$ $\quad E[X^2] = \int_{-\infty}^{\infty} x^2 f_x(x)dx$ = mean square value = M_2 = MSV.

(ii) Central Moment:

$$E[(X-m_x)^n] = \int_{-\infty}^{\infty} (x-m_x)^n f_x(x)dx$$

If $n = 1$ $$E[X-m_x] = \int_{-\infty}^{\infty} (x-m_x) f_x(x)dx$$

$$= \int_{-\infty}^{\infty} X b_x(x)d_x - E\int_{-\infty}^{\infty} m_x . f_x(x)d_x$$

$$= E\left[\int_{-\infty}^{\infty} x.f_X(x)d_x\right] - E\left[\int_{-\infty}^{\infty} m_x.f_X(x)d_x\right]$$

$$= m_x - m_x = 0$$

If $n = 2$ $\quad E[(X-m_x)^2] = \int_{-\infty}^{\infty} (x-m_x)^2 f_X(x)\, d_x$

$$\sigma_x^2 = E\,[(x-m_x)^2]$$

$$= E[x^2 + m_x^2 - 2Xm_x]$$

$$= E\,[X^2] + E\,[m_x^2] - 2m_x E[x]$$

$$= E\,[X^2] + m_x^2 - 2m_x\, E[x]$$

$$= E\,[X^2] + m_x^2 - 2\, m_x\,.m_x$$

$$= E[X^2] - m_x^2$$

Variance $\quad \sigma_x^2 = E[X^2] - E^2[X] = M_2 - M_1^2$

Hence, Standard deviation

$$\sigma_x = \sqrt{\sigma_x^2}$$

10. CONCLUSION

(1) $E[X] = \int_{-\infty}^{\infty} x\, f_x(x)\, dx$

(2) $E[X^2] = \int_{-\infty}^{\infty} x^2 f_x(x)\, dx$

(3) $\sigma_x^2 = E[X^2] - E^2[X]$

(4) $\sigma_x = \sqrt{\sigma_x^2}$

Note: *If X and Y are two statistically independent variable then*

$$E[X + Y] = E[X] + E[Y]$$

$$E[(X + Y)^2] = E[X^2] + E[Y^2]$$

$$\sigma_{(x+y)}^2 = \sigma_x^2 + \sigma_Y^2$$

$$E[XY] = E[X].\,E[Y]$$

11. RANDOM PROCESS:

Random variable is the mapping from sample space to real axis where as random process is the mapping from sample space to the waveform which is a function of time.

Random process is basically a collection of all the waveforms where all the waveforms are known in advance but the exact value is not known.

12. CLASSIFICATION OF RANDOM PROCESS

(i) Stationary Random Process :

A random process can be characterized by joint PDF and joint CDF of all the random variable obtained by sampling the random process at different time. A random process whose statistical characteristic do not change with time is known as stationary random process and in such process, time shift does not affect the characteristic of the process

(ii) Wide Sense Stationary Random Process :

A Random process is said to be a WSSRP when

(a) Mean value is constant

(b) Auto-Correlation

$$R_X(\tau) = E[X(t_1).X(t_2)] = \text{function of } (t_2 - t_1) \text{ only}$$

In Auto-Correlation, same function is sampled at two different or same instant of time.

Auto-Correlation refers to the correlation of two random variables obtained by sampling the same random process at two different instant of time.

13. PROPERTIES:

(1) $RX(\tau) = E[X(t_1).X(t_2)]$

$$t_1 = t \quad ; t$$
$$t_2 = t + \tau \;\; ; \;\; t - \tau$$
$$R_X(\tau) = E[X(t).X(t+\tau)]$$
$$= E[X(t).X(t-\tau)]$$

(2) Auto-Correlation function is even function

$$R_X(\tau) = R_X(-\tau)$$

(3) All the same instant of time, Auto-Correlation function is maximum

$$R_X(\tau) \le R_X(0)$$

(4) $$\boldsymbol{R_X(0) = E[x(t).x(t)]}$$
$$= E[X^2]$$
$$= \text{M.S.V. (Mean square value)}$$

14. POWER SPECTRAL DENSITY :

For deterministic signal, in case of frequency domain, we talk in terms of amplitude and frequency. But for random process it is meaning less to talk about fourier transform of different waveforms corresponding to different sample point.

For obtaining the frequency domain description of random process, concept of power spectral density is used which gives an idea how much power is contained in the frequency of the given signial

15. DEFAULT (WSSRP)

PSD: $S_X(f)$ = input power spectral density

$S_Y(f)$ = Output power spectral density

To find power spectral density of any random process, we require fourier transform a relation between spectral density and auto-correlation function

Mathematically: $S_X(f) = \text{F.T}\ \{R_X(\tau)\}$

$R_X(\tau) = \text{I. F. T. } \{S_X(f)\}$

Here, F.T. = Fouries Transform

Similarly $S_Y(f) = \text{F.T.}\{R_Y(\tau)\}$

$R_Y(\tau) = \text{I.F.T.}\{S_Y(f)\}$

Here, I.F.T. = Inverse fouries transform

In frequency domain,

$$S_X(f) = \text{F.T.}\{R_X(\tau)\}$$

$$\Rightarrow \quad S_X(f) = \int_{-\infty}^{\infty} R_x(\tau) e^{-j2\pi f\tau} d\tau$$

$$R_X(\tau) = \int_{-\infty}^{\infty} S_x(f).e^{-j2\pi f\tau} df$$

16. PROPERTIES OF PSD

(i) $S_X(f) \geq 0$; for all frequency

(ii) $S_X(f) = S_X(-f)$

i.e. even symmetry

(iii) $E[X^2] = \int_{-\infty}^{\infty} S_X(f)\,df = R_X(0)$ = Area under the curve

(iv) PDF of PSD

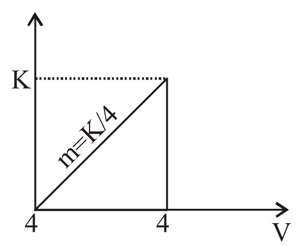
$$P_X(x) = \frac{S\times(f)}{\int_{-\infty}^{\infty} S_X(f)\,df}$$

$$0 \leq P_X(x) \leq 1$$

Example: An output of a communication channel is a random variable V with the probability density function as shown. The mean square value of V is ?

Solution: $P(V) = \frac{K}{4}\ ;\ 0 \leq V \leq 4$

$= 0\ ;$ elsewhere



$$E[V^2] = \int_{-\infty}^{\infty} V^2 P(V)\,dV$$

We know that $\int_{-\infty}^{\infty} P(V)\,dV = 1$

$$\Rightarrow \quad \frac{1}{2} \times 4 \times K = 1$$

$$K = \frac{1}{2}$$

$$E[V^2] = \int_{-\infty}^{\infty} V^2 \frac{V}{8} dV = \int_{0}^{4} \frac{V^3}{8}\,dV = \left[\frac{V^4}{32}\right]_0^4$$

$$= \frac{1}{32} \times 4^4 = 8$$

17. PROBABILITY MODELS

It has been derived for various random variable for their analysis.

(i) Binomial Distribution function.

(ii) Poison Distribution function.

(iii) Gaussion Distribution function.

(iv) Uniform Distribution function.

(v) Rayleigh Distribution function

Here, Binomial & Poison are defined for discrete random variables and remaining function are defined for continuous random variable.

(i) Bionomial Distribution Function :

It is applied to discrete random variables assigned integer values associated with repeated trials of an experiment. Therefore, this function can be applied to find out number of errors in a message of n digits.

Let K be the number of message bits set n correctly out of n number of bits, then (n-k), no of bits will be have error. The probability of having K correct bits in a word is given by binomial distribution function.

PDF for Binomial distribution function is defined by

$$p_X(X) = \sum_{K=0} nC_k p^k (1-p)^{n-k} \delta(X-k)$$

C.D.F. for binomial distribution is given by

$$F_X(x) = \sum_{k=0}^{x} nC_k p^k (1-p)^{n-k} = P(x \le x)$$

p = bit error probability

Note: *CDF for binomial distribution is used for obtaining probability of error during transmisson of bits over a communication channel*

Example: During transmission of data over a communication channel bit errors occur independently with probability p if block of n bits to be transmtted. Then probability at the most one bit error occurs during transmission is

Solution:

$$p(e \le 1) = \sum_{k=0}^{x} nC_k p^k (1-p)^{n-k}$$

$$= n\,C_0 P^0\ (1-p)^{n-0} + nC_1 P^1 (1-P)^{n-1}$$

$$p(e \le 1) = (1-P)^n + nP\ (1-P)^{n-1}$$

Mean value of binomial distribtuion.

$$m_x = n.p$$

$p \rightarrow$ probabilty of occurance

$n \rightarrow$ total no of occurance

Variance $\qquad V_x = n\,p\,(1-p)$

$$\sigma_x = \sqrt{np(n-p)}$$

(ii) Poisson Distribution Function:

This is another variable distribution function used for discrete random variable.

Probability of occurance of an event K-times can be defined by poison distrubution function as undre.

$$P(x = K) = \frac{k^k e^{-m}}{\lfloor k}$$

Where $\qquad m = np$; $n \rightarrow$ total number of events.

Variance $\qquad Vx = \sigma_x^2 = np$

$$\sigma_x = \sqrt{np}$$

Note: *Poisson distribution is preferred over binomial when n is very large and poison distribtution is limiting case of binomial distrigtuion.*

(iii) Gaussion Distribution Function:

It is used for continuous random variable. It is most impotant distribtuion for communication system. Majority of noise in communication system has Gaussion distribtuion. Gaussion PDF for a continuous random variable is defined by

$$p_x(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \cdot e^{-(x-m)^2/2\sigma^2}$$

Where $\qquad m$ = mean value

σ = standard deviation.

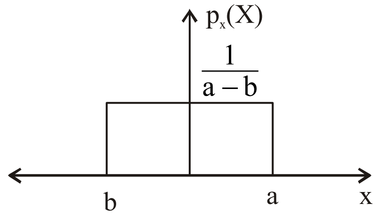
Gaussion CDF $\quad F_x(X) = \int_{-\infty}^{x} P_x(\lambda).d\lambda$

Note: *CDF of gaussion PDF is a error function.*

(iv) Uniform Distribution Function:

If a continuous random variable has equal porbability of occurance over a finite range and zero valued outside that range then ramdom variable is said to have uniform distribution.

Uniform PDF for such variable



$$p_x(X) = \frac{1}{a-b}\ ;\ a \le x \le b$$

$$= 0\ ;\ \text{else where}$$

Note: *Uniform PDF is used for find out quantization error in PCM system & Grainular noise in delta modulation.*

$$\text{RMS} = \int_{b}^{a} x^2 p_x(x)dx$$

mean value $\quad = \int_{b}^{a} x^2 p_x(x)dx$

(iv) Rayleigh Distribution Function:

18. RANDOM SIGNALS

(i) Power Spectrum of Random Signals:

When a random signal v(t) is stationary then its power spectrum is distribution of average power over frequency domain. The power spectral density of a random signal is equal to the Fourier transform of auto correlation function the signal.

∴ For a random signal v(t)

$$G_v(f) = f\{R(\tau)\} = \int_{-\infty}^{\infty} R(\tau)e^{-j2\pi f\tau}\,d\tau$$

∴ Auto correlation function will be

$$R(\tau) = f^{-1}\{G_v(f)\} = \int_{-\infty}^{\infty} G_v(f)e^{-j2\pi f\tau}\,df$$

Note: *Autor correlation function and spectral density of random signals constitute Fourier transofrm pair just like deterministic signal.*

Properties of spectral density:

(a) $\int_{-\infty}^{\infty} G_v(f)df = \overline{V^2}(t) = P = R(0)$

$G_v(f) \rightarrow$ spectral density or v(t)

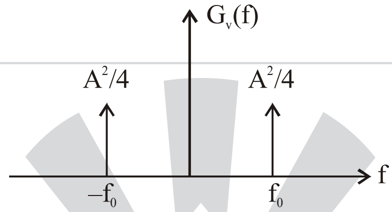
(b) $G_v(f) \geq 0$

(c) $G_v(f) = G_v(-f) \rightarrow$ even function

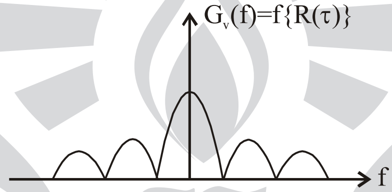
> ***Note:*** *The power spectrum of random process may be continuous, impulse or mixed depending upon the source*
>
> *For example:-For a sinusoidal signal with initial random phase the spectrum is two impulses.*

$v(t) = A \sin(\omega t + \phi)$; $\phi \rightarrow$ Initial random phase.

$\therefore \quad G_v(f) = f\{R(\tau)\}$

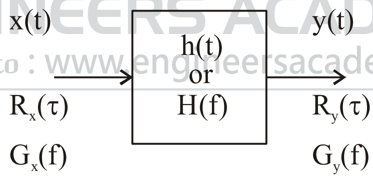


For a random digital wave, the power spectrum is continuous wave



(ii) Filtering of Random Signals :

Let a random signal is passed through a LTI filter as shown below



Let x(t) is a stationary power then y(t) would also be stationary power signal

then $\quad R_y(\tau) = h(\tau) * R_x(\tau)$

and similarly $\quad R_y(\tau) = h(-\tau) * R_{yx}(\tau)$

$\therefore \quad G_v(f) = |H(f)|^2 G_x(f)$

Then, the spectral density of output and input signal.

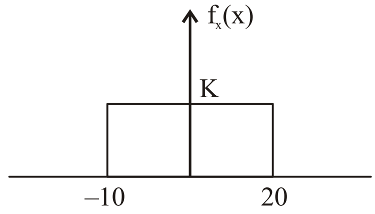
$$G_y(f) = |H(f)|^2 G_x(f)$$

$$R_y(\tau) = f^{-1}\{|H(f)|^2 G_x(f)\}$$

$$R_y(\tau) = \int_{-\infty}^{\infty} |H(f)|^2\, G_x(f) e^{+j2\pi f\tau} df$$

$$R_y(0) = \int_{-\infty}^{\infty} |H(f)|^2\, df = \text{output power}$$

Example: For a continous random variable 'x' Density if is given below. Find



(i) K. (ii) P $(-10 \le x \le 10)$

(iii) P $(x \le 10)$ (iv) Plot $F_X(x)$

Solution: ***(i)***

$$\int_{-\infty}^{\infty} f_x(x)\,dx = 1$$

$$30K = 1$$

$$K = 1/30$$

(ii)

$$P(-10 \le x \le 10) = \int_{-10}^{10} f_x(x)dx = \frac{1}{30} \times 20 = \frac{2}{3}$$

(iii)

$$P(x \ge 10) = 1 - \frac{2}{3} = \frac{1}{3}$$

$$= \int_{10}^{\infty} f_x(x)dx = \int_{10}^{20} f_x(x)dx = \frac{10}{30} = \frac{1}{3}$$

(iv)

$$F_x(x) = \int_{-\infty}^{x} f_x(x)dx = \int_{-10}^{x} \frac{1}{30}dx = \frac{x+10}{30}\ [-10, 20]$$

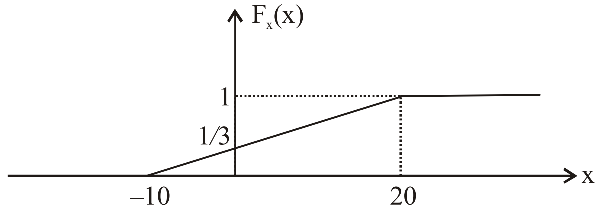
$$F_x(-20) = \frac{-20+10}{30} = \frac{-1}{3}.$$ But ......$0 \le F_x(x) \le 1$

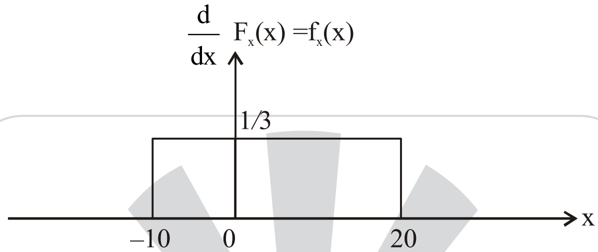
$$F_x(-20) = P(x \le -20) = \int_{-\infty}^{-20} f_x(x)dx = \underline{\underline{0}}$$

$$F_x(30) = \frac{4}{3} \text{ wrong } \because\ F_x(x) \le 1$$

$$F_x(30) = P(x \le 30) = \int_{-\infty}^{30} f_x(x)dx = 1$$

Hence,
$$F_x(x) = \begin{cases} 0 & ; x \le -10 \\ \dfrac{x+10}{30} & ; -10 \le x \le 20 \\ 1 & ; x \ge 20 \end{cases}$$





Example: For a continous random variable 'X'

$f_x(x) = ae^{-bx} \quad ; x \ge 0$

$0 \quad ; \text{ else where}$

(i) Find relation between a and b

(ii) Find the probability $P\,(1 \le x \le 5)$

(iii) Plot $F_x(x)$

Solution: ***(i)***
$$\int_{-\infty}^{\infty} f_x(x)dx = 1 \Rightarrow \int_{-\infty}^{\infty} ae^{-bx}dx = 1$$

$$\left[\frac{ae^{-bx}}{-b}\right]_0^{\infty} = 1 \Rightarrow \frac{a}{b} = 1 \Rightarrow a = b$$

$$f_x(x) = \begin{cases} ae^{-ax} & ; \quad x \ge 0 \\ 0 & ; \text{ otherwise} \end{cases}$$

(ii)
$$P(1 \le x \le 5) = \int_1^5 f_x(x)dx = \left.\frac{ae^{-ax}}{-a}\right|_1^5 = e^{-a}e^{-5a}$$

(iii)
$$F_x(x) = \int_{1\;0}^{5\;x} f_x(x)dx = \int_0^x e^{-ax}a\,dx = 1 - e^{-ax}$$

$$F_x(x) = \begin{cases} 1 - e^{-ax} & x \ge 0 \\ 0 & x < 0 \end{cases}$$